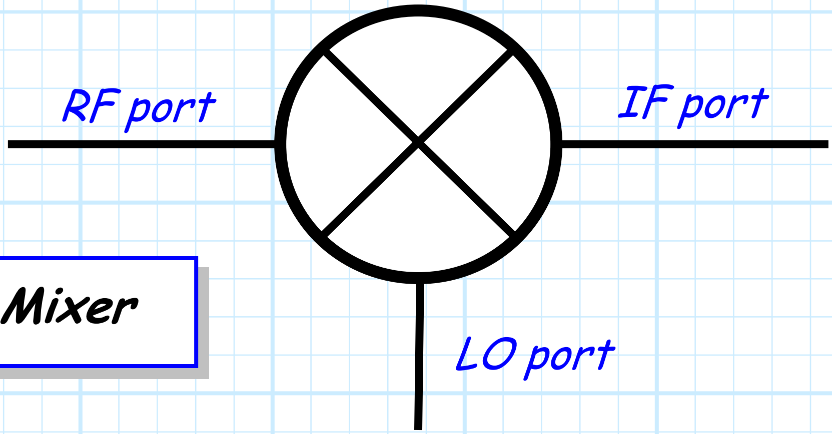
Mixers

A **mixer** is a three-port, **non-linear** microwave device.



The Mixer

The three ports of a mixer are **distinct** and unique, and are typically referred to as:

1) The **RF** (Radio Frequency) port

2) The **IF** (Intermediate Frequency) port

3) The **LO** (Local Oscillator) port

Q: So just what does a mixer **do**??

A: A **clue** is in its **symbol**: $\otimes$

→ A mixer is a **multiplier** ($\times$) !!

Say there is a signal $v_{RF}(t)$ at the RF mixer port, and a signal $v_{LO}(t)$ at the LO mixer port. An **ideal** mixer would then produce at the IF port, a signal $v_{IF}(t)$, where:

$$v_{RF}(t)\, v_{LO}(t) = v_{IF}(t) \quad \text{(an \textbf{ideal} mixer)}$$

To see why this might be **useful**, consider a case where:

$$v_{RF}(t) = \cos \omega_{RF} t$$

$$v_{RF}(t) = \cos \omega_{LO} t$$

Multiplying these signals, we get:

$$\begin{aligned} v_{IF}(t) &= v_{RF}(t)\, v_{LO}(t) \\ &= \cos(\omega_{RF} t) \cos(\omega_{LO} t) \\ &= \frac{1}{2} \cos(\omega_{RF} - \omega_{LO}) t + \frac{1}{2} \cos(\omega_{RF} + \omega_{LO}) t \end{aligned}$$

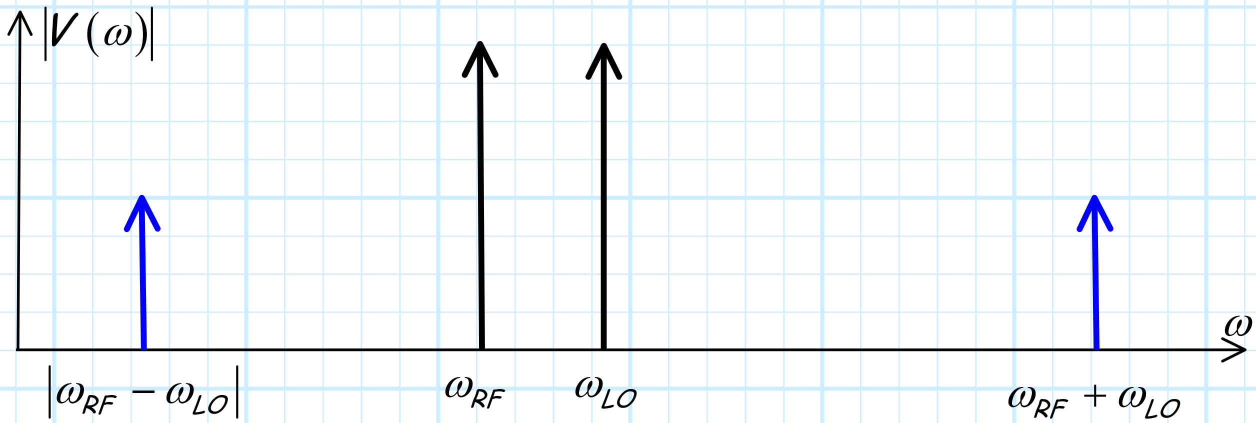
At the IF port we have created **two** signals with **new** frequencies!

One new signal has a frequency that is the **difference** of the LO and RF signal frequencies:

$$\frac{1}{2} \cos(\omega_{RF} - \omega_{LO}) t$$

While the other new signal has a frequency that is the **sum** of the LO and RF signal frequencies:

$$\frac{1}{2}\cos\left(\omega_{RF}+\omega_{LO}\right)t$$



But alas, mixers are **not ideal**!

A more **accurate** model of the (**non**-ideal) relationship between $v_{RF}(t)$, $v_{LO}(t)$, and $v_{IF}(t)$ is:

$$\begin{aligned} v_{IF}(t) = {} & a_1\, v_{RF}(t) + a_2\, v_{LO}(t) \\ & + a_3\, v_{RF}^2(t) + a_4\, v_{RF}(t) v_{LO}(t) + a_5\, v_{LO}^2(t) \\ & + a_6\, v_{RF}^3(t) + a_7\, v_{RF}^2(t) v_{LO}(t) \\ & + a_8\, v_{RF}(t) v_{LO}^2(t) + a_9\, v_{LO}^3(t) \\ & + \cdots \end{aligned}$$

where the values a_n are real-valued **constants**.

Just as with an amplifier, a mixer will produce **1st-order**, **2nd-order**, **3rd-order**, and even **higher order** terms!

As a result, there will be **many** signals created at the IF port. If we did **all** the trigonometry, we would find that the signal **frequencies** created from these terms (in relation to ω_{RF} and ω_{LO}) are:

1st order: ω_{RF}, ω_{LO}

2nd order: $\left|\omega_{RF} - \omega_{LO}\right|, 2\omega_{RF}, 2\omega_{LO}, \omega_{RF} + \omega_{LO}$

3rd order: $\left|2\omega_{RF} - \omega_{LO}\right|, \left|2\omega_{LO} - \omega_{RF}\right|, 3\omega_{RF},$
$3\omega_{LO}, 2\omega_{RF} + \omega_{LO}, \omega_{RF} + 2\omega_{LO}$

examples of higher orders: $\left|4\omega_{RF} - 2\omega_{LO}\right|, 5\omega_{RF},$
$7\omega_{LO}, 827\omega_{RF} + 134\omega_{LO}$

Note that the **ideal** mixer (multiplier) occurs when all constants a_n are zero, **except** for the constant a_4. The result in this case being:

$$v_{IF}(t) = a_4\, v_{RF}(t)\, v_{LO}(t) \qquad \text{(ideal mixer response)}$$

and thus the **only** signals created are the 2nd order terms $\left|\omega_{RF} - \omega_{LO}\right|$ and $\omega_{RF} + \omega_{LO}$.

Of course for a "real" mixer, all the constants a_n are **non-zero**, although for good mixers all but a_4 are relatively **small**.

Thus, for good mixers, most of the signals created at the IF output will be of relatively **low** power, with **exception** of the signal at frequencies $|\omega_{RF} - \omega_{LO}|$ and $\omega_{RF} + \omega_{LO}$.

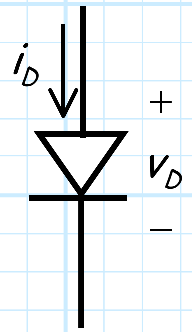
All **other** signals (meaning other than $|\omega_{RF} - \omega_{LO}|$ and $\omega_{RF} + \omega_{LO}$) at the IF are known as **spurious** signals—or in the vernacular of radio engineers, "**spurs**".

Q: *How are mixers* ***constructed****?*

A: Multiplication is a decidedly **non-linear** operation. As such, it requires **non-linear devices** to implement.

Typically, these non-linear devices are **diodes**, but sometimes transistors are used.

For example, as those of you who aced EECS 312 know, the **junction diode** equation is:



$$i_D = I_s \left(e^{v_D / nV_T} - 1 \right)$$

This **non-linear** function can be expanded using a **Taylor series** as:

$$i_D = I_s \left(e^{v_D / nV_T} - 1 \right) = b_1 v_D + b_2 v_D^2 + b_3 v_D^3 + \cdots$$

And if, for example:

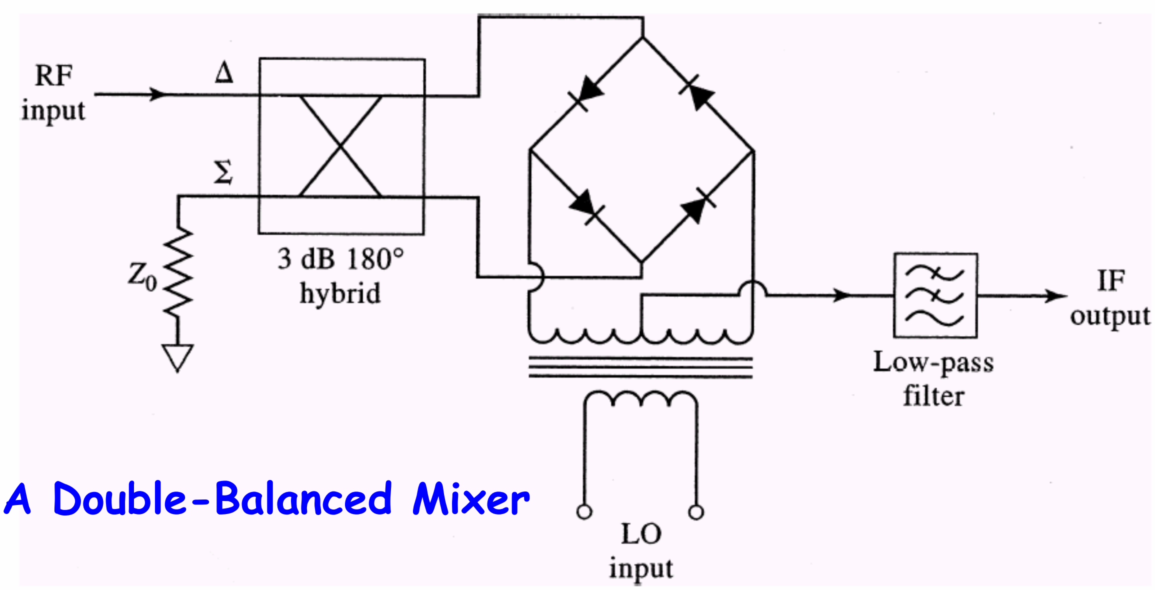
$$v_D(t) = v_{RF}(t) + v_{LO}(t)$$

we find that the diode will create **high-order** terms, including $v_{RF}(t)v_{LO}(t)$:

$$b_2\left(v_{RF}(t) + v_{LO}(t)\right)^2 = b_2\left(v_{RF}^2(t) + 2v_{RF}(t)v_{LO}(t) + v_{LO}^2(t)\right)$$

Basically, generating high-order terms with **any** non-linear device is **not** at all difficult (just try and **keep** it from happening!). The trick is to generate **only** the 2nd order term $v_{RF}(t)v_{LO}(t)$, while somehow **suppressing** the rest.

Thus, mixer design is as much **art** as it is science! Popular designs include the **balanced mixer** (with 2 junction diodes), and the **double balanced mixer** (with 4 junction diodes).



A Double-Balanced Mixer